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PAIRWISE SEMIOPEN AND SEMICLOSED MAPPINGS IN INTUITIONISTIC SMOOTH BITOPOLOGICAL SPACES

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Abstract. In this paper, we introduce intuitionistic fuzzy pairwise semiopen and semiclosed mappings in intuitionistic smooth bitopological spaces, and obtain the characterizations for the mappings.

1. Introduction and preliminaries

In the previous paper [6], the authors introduced the concepts of intuitionistic smooth bitopological spaces and two kinds of continuity in the spaces. In this paper, we introduce intuitionistic fuzzy pairwise semiopen and semiclosed mappings in intuitionistic smooth bitopological spaces, and obtain the characterizations for the mappings.

Throughout this paper, I denotes the unit interval $[0,1]$ of the real line and $I_0 = (0,1]$. A member μ of I^X is called a *fuzzy set* in X. For any $\mu \in I^X$, μ^c denotes the complement $1-\mu$. By $\tilde{0}$ and $\tilde{1}$ we denote constant mappings on X with value of 0 and 1, respectively.

Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of nonmembership, respectively, and $\mu_A + \gamma_A \le 1$. Obviously, every fuzzy set μ in X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1}-\mu)$. $I(X)$ denotes a family of all intuitionistic fuzzy sets in X and "IF" stands for intuitionistic fuzzy.

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All the definitions and notations which are not mentioned in this paper, we refer to [6].

Definition 1.1 ([6]). An *intuitionistic smooth topology* on X is a mapping $\mathcal{T} : I(X) \to I$ which satisfies the following properties:

(1) $\mathcal{T}(\underline{0}) = \mathcal{T}(\underline{1}) = 1$.
(2) $\mathcal{T}(A \cap B) \geq \mathcal{T}(A) \wedge \mathcal{T}(B)$.
(3) $\mathcal{T}(\bigvee A_i) \geq \bigwedge \mathcal{T}(A_i)$.

The pair $(X, \mathcal{T})$ is called an *intuitionistic smooth topological space*.

Definition 1.2 ([6]). A system $(X, \mathcal{T}_1, \mathcal{T}_2)$ consisting of a set X with two intuitionistic smooth topologies $\mathcal{T}_1$ and $\mathcal{T}_2$ on X is called a *intuitionistic smooth bitopological space*(ISBTS for short). Throughout this paper the indices i, j take the value in $\{1, 2\}$ and $i \neq j$.

Definition 1.3 ([6]). Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from an ISBTS X to an ISBTS Y and $r, s \in I_0$. Then f is said to be *IF pairwise (r, s)-continuous* if the induced mapping $f : (X, \mathcal{T}_1) \to (Y, \mathcal{U}_1)$ is an IF r-continuous mapping and the induced mapping $f : (X, \mathcal{T}_2) \to (Y, \mathcal{U}_2)$ is an IF s-continuous mapping.

Definition 1.4 ([6]). Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from an ISBTS X to an ISBTS Y and $r, s \in I_0$. Then f is said to be *IF pairwise (r, s)-semicontinuous* if $f^{-1}(A)$ is an IF $(\mathcal{T}_1, \mathcal{T}_2)$-$(r, s)$-semiopen set in X for each IF $\mathcal{U}_1$-r-open set A in Y and $f^{-1}(B)$ is an IF $(\mathcal{T}_2, \mathcal{T}_1)$-$(s, r)$-semiopen set in X for each IF $\mathcal{U}_2$-s-open set B in Y.

2. Intuitionistic fuzzy pairwise semiopen and semiclosed mappings

Now we define the concepts of IF pairwise (r, s)-semiopen and semiclosed mappings in intuitionistic smooth bitopological spaces, and investigate some of their properties.

Definition 2.1. Let $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ be a mapping from an intuitionistic smooth topological space X to an intuitionistic smooth topological space Y and $r \in I_0$. Then f is called

(1) an *IF r-open* mapping if $f(A)$ is IF $\mathcal{U}$-r-open in Y for each IF $\mathcal{T}$-r-open set A in X,
(2) an *IF r-closed* mapping if $f(A)$ is IF $\mathcal{U}$-r-closed in Y for each IF $\mathcal{T}$-r-closed set A in X.

Definition 2.2. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from an ISBTS X to an ISBTS Y and $r, s \in I_0$. Then f is said to be

(1) *IF pairwise* (r, s)*-open* if the induced mapping $f : (X, \mathcal{T}_1) \to (Y, \mathcal{U}_1)$ is an IF r-open mapping and the induced mapping $f : (X, \mathcal{T}_2) \to (Y, \mathcal{U}_2)$ is an IF s-open mapping,
(2) *IF pairwise* (r, s)*-closed* if the induced mapping $f : (X, \mathcal{T}_1) \to (Y, \mathcal{U}_1)$ is an IF r-closed mapping and the induced mapping $f : (X, \mathcal{T}_2) \to (Y, \mathcal{U}_2)$ is an IF s-closed mapping.

Definition 2.3. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from an ISBTS X to an ISBTS Y and $r, s \in I_0$. Then f is said to be

(1) *IF pairwise* (r, s)*-semiopen* if $f(A)$ is IF $(\mathcal{U}_1, \mathcal{U}_2)$-$(r, s)$-semiopen in Y for each IF $\mathcal{T}_1$-r-open set A in X and $f(B)$ is IF $(\mathcal{U}_2, \mathcal{U}_1)$-$(s, r)$-semiopen in Y for each IF $\mathcal{T}_2$-s-open set B in X,
(2) *IF pairwise* (r, s)*-semiclosed* if $f(A)$ is IF $(\mathcal{U}_1, \mathcal{U}_2)$-$(r, s)$-semiclosed in Y for each IF $\mathcal{T}_1$-r-closed set A in X and $f(B)$ is IF $(\mathcal{U}_2, \mathcal{U}_1)$-$(s, r)$-semiclosed in Y for each IF $\mathcal{T}_2$-s-closed set B in X.

Remark 2.4. It is clear that every IF pairwise (r, s)-open mapping is IF pairwise (r, s)-semiopen. But the following example shows that the converse need not be true.

Example 2.5. *Let* $X = \{x, y\}$ *and let* A_1, A_2, A_3, *and* A_4 *be intuitionistic fuzzy sets in* X *defined as*

$$A_1(x) = (0.1, 0.7),\ A_1(y) = (0.7, 0.2);$$

$$A_2(x) = (0.6, 0.2),\ A_2(y) = (0.3, 0.6);$$

$$A_3(x) = (0.1, 0.7),\ A_3(y) = (0.9, 0.1);$$

and

$$A_4(x) = (0.7, 0.1),\ A_4(y) = (0.3, 0.6).$$

Define $\mathcal{T}_1 : I(X) \to I$ *and* $\mathcal{T}_2 : I(X) \to I$ *by*

$$\mathcal{T}_1(A) = \begin{cases} 1 & \text{if } A = \underline{0}, \underline{1}, \\ \frac{1}{2} & \text{if } A = A_1, \\ 0 & \text{otherwise;} \end{cases}$$

and

$$\mathcal{T}_2(A) = \begin{cases} 1 & \text{if } A = \underline{0}, \underline{1}, \\ \frac{1}{3} & \text{if } A = A_2, \\ 0 & \text{otherwise.} \end{cases}$$

Define $\mathcal{U}_1 : I(X) \to I$ *and* $\mathcal{U}_2 : I(X) \to I$ *by*

$$\mathcal{U}_1(A) = \begin{cases} 1 & \text{if } A = \underline{0}, \underline{1}, \\ 0 & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}_2(A) = \begin{cases} 1 & \text{if } A = \underline{0}, \underline{1}, \\ \frac{1}{3} & \text{if } A = A_4, \\ 0 & \text{otherwise.} \end{cases}$$

Then $(X, \mathcal{T}_1, \mathcal{T}_2)$ *and* $(X, \mathcal{U}_1, \mathcal{U}_2)$ *are ISBTSs. Consider a mapping* $g : (X, \mathcal{U}_1, \mathcal{U}_2) \to (X, \mathcal{T}_1, \mathcal{T}_2)$ *defined by* $g(x) = x$ *and* $g(y) = y$*. Then* g *is an IF pairwise* $(\frac{1}{2}, \frac{1}{3})$*-semiopen mapping. But* g *is not IF pairwise* $(\frac{1}{2}, \frac{1}{3})$*-open.*

THEOREM 2.6. *Let* $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ *be a mapping from an ISBTS* X *to an ISBTS* Y *and* $r, s \in I_0$*. Then the following statements are equivalent:*

(1) f *is IF pairwise* (r, s)*-semiopen.*
(2) *For each intuitionistic fuzzy set* A *in* X,

$$f(\mathcal{T}_1\text{-}int(A, r)) \subseteq (\mathcal{U}_1, \mathcal{U}_2)\text{-}sint(f(A), r, s)$$

and

$$f(\mathcal{T}_2\text{-}int(A, s)) \subseteq (\mathcal{U}_2, \mathcal{U}_1)\text{-}sint(f(A), s, r).$$

(3) *For each intuitionistic fuzzy set* B *in* Y,

$$\mathcal{T}_1\text{-}int(f^{-1}(B), r) \subseteq f^{-1}((\mathcal{U}_1, \mathcal{U}_2)\text{-}sint(B, r, s))$$

and

$$\mathcal{T}_2\text{-}int(f^{-1}(B), s) \subseteq f^{-1}((\mathcal{U}_2, \mathcal{U}_1)\text{-}sint(B, s, r)).$$

Proof. (1) $\Rightarrow$ (2) Let A be an intuitionistic fuzzy set in X. Then $\mathcal{T}_1$-int(A, r) is an IF $\mathcal{T}_1$-r-open set and $\mathcal{T}_2$-int(A, s) is an IF $\mathcal{T}_2$-s-open set in X. Since f is IF pairwise (r, s)-semiopen, $f(\mathcal{T}_1\text{-int}(A, r))$ is IF $(\mathcal{U}_1, \mathcal{U}_2)$-$(r, s)$-semiopen and $f(\mathcal{T}_2\text{-int}(A, s))$ is IF $(\mathcal{U}_2, \mathcal{U}_1)$-$(s, r)$-semiopen in Y. Thus

$$\begin{aligned} f(\mathcal{T}_1\text{-int}(A, r)) &= (\mathcal{U}_1, \mathcal{U}_2)\text{-sint}(f(\mathcal{T}_1\text{-int}(A, r)), r, s) \\ &\subseteq (\mathcal{U}_1, \mathcal{U}_2)\text{-sint}(f(A), r, s) \end{aligned}$$

and

$$\begin{aligned} f(\mathcal{T}_2\text{-int}(A, s)) &= (\mathcal{U}_2, \mathcal{U}_1)\text{-sint}(f(\mathcal{T}_2\text{-int}(A, s)), s, r) \\ &\subseteq (\mathcal{U}_2, \mathcal{U}_1)\text{-sint}(f(A), s, r). \end{aligned}$$

(2) $\Rightarrow$ (3) Let B be an intuitionistic fuzzy set in Y. Then by (2), we have

$$\begin{aligned} f(\mathcal{T}_1\text{-int}(f^{-1}(B),r)) &\subseteq (\mathcal{U}_1,\mathcal{U}_2)\text{-sint}(f(f^{-1}(B)),r,s) \\ &\subseteq (\mathcal{U}_1,\mathcal{U}_2)\text{-sint}(B,r,s) \end{aligned}$$

and

$$\begin{aligned} f(\mathcal{T}_2\text{-int}(f^{-1}(B),s)) &\subseteq (\mathcal{U}_2,\mathcal{U}_1)\text{-sint}(f(f^{-1}(B)),s,r) \\ &\subseteq (\mathcal{U}_2,\mathcal{U}_1)\text{-sint}(B,s,r). \end{aligned}$$

Hence

$$\mathcal{T}_1\text{-int}(f^{-1}(B),r) \subseteq f^{-1}((\mathcal{U}_1,\mathcal{U}_2)\text{-sint}(B,r,s))$$

and

$$\mathcal{T}_2\text{-int}(f^{-1}(B),s) \subseteq f^{-1}((\mathcal{U}_2,\mathcal{U}_1)\text{-sint}(B,s,r)).$$

(3) $\Rightarrow$ (1) Let A be any IF $\mathcal{T}_1$-r-open set and B any IF $\mathcal{T}_2$-s-open set in X. Then $A = \mathcal{T}_1\text{-int}(A,r)$ and $B = \mathcal{T}_2\text{-int}(B,s)$. By (3), we obtain

$$\begin{aligned} A = \mathcal{T}_1\text{-int}(A,r) &\subseteq \mathcal{T}_1\text{-int}(f^{-1}(f(A)),r) \\ &\subseteq f^{-1}((\mathcal{U}_1,\mathcal{U}_2)\text{-sint}(f(A),r,s)) \end{aligned}$$

and

$$\begin{aligned} B = \mathcal{T}_1\text{-int}(B,s) &\subseteq \mathcal{T}_2\text{-int}(f^{-1}(f(B)),s) \\ &\subseteq f^{-1}((\mathcal{U}_2,\mathcal{U}_1)\text{-sint}(f(B),s,r)). \end{aligned}$$

Thus

$$f(A) \subseteq (\mathcal{U}_1,\mathcal{U}_2)\text{-sint}(f(A),r,s)$$

and

$$f(B) \subseteq (\mathcal{U}_2,\mathcal{U}_1)\text{-sint}(f(B),s,r).$$

Hence

$$f(A) = (\mathcal{U}_1,\mathcal{U}_2)\text{-sint}(f(A),r,s)$$

and

$$f(B) = (\mathcal{U}_2,\mathcal{U}_1)\text{-sint}(f(B),s,r).$$

Thus $f(A)$ is an IF $(\mathcal{U}_1,\mathcal{U}_2)$-$(r,s)$-semiopen set and $f(B)$ is an IF $(\mathcal{U}_2,\mathcal{U}_1)$-$(s,r)$-semiopen set in Y. Therefore f is an IF pairwise (r,s)-semiopen mapping. □

THEOREM 2.7. *Let $f : (X,\mathcal{T}_1,\mathcal{T}_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a mapping from an ISBTS X to an ISBTS Y and $r,s \in I_0$. Then the following statements are equivalent:*

(1) *f is IF pairwise (r,s)-semiclosed.*

(2) *For each intuitionistic fuzzy set A in X,*

$$(\mathcal{U}_1,\mathcal{U}_2)\text{-}scl(f(A),r,s) \subseteq f(\mathcal{T}_1\text{-}cl(A,r))$$

and

$$(\mathcal{U}_2,\mathcal{U}_1)\text{-}scl(f(A),s,r) \subseteq f(\mathcal{T}_2\text{-}cl(A,s)).$$

Proof. (1) $\Rightarrow$ (2) Let A be an intuitionistic fuzzy set in X. Then $\mathcal{T}_1$-cl(A,r) is IF $\mathcal{T}_1$-r-closed and $\mathcal{T}_2$-cl(A,s) is IF $\mathcal{T}_2$-s-closed in X. Since f is IF pairwise (r,s)-semiclosed, $f(\mathcal{T}_1\text{-cl}(A,r))$ is IF $(\mathcal{U}_1,\mathcal{U}_2)$-$(r,s)$-semiclosed and $f(\mathcal{T}_2\text{-cl}(A,s))$ is IF $(\mathcal{U}_2,\mathcal{U}_1)$-$(s,r)$-semiopen in Y. Hence

$$\begin{aligned}(\mathcal{U}_1,\mathcal{U}_2)\text{-scl}(f(A),r,s) &\subseteq (\mathcal{U}_1,\mathcal{U}_2)\text{-scl}(f(\mathcal{T}_1\text{-cl}(A,r)),r,s)\\ &= f(\mathcal{T}_1\text{-cl}(A,r))\end{aligned}$$

and

$$\begin{aligned}(\mathcal{U}_2,\mathcal{U}_1)\text{-scl}(f(A),s,r) &\subseteq (\mathcal{U}_2,\mathcal{U}_1)\text{-scl}(f(\mathcal{T}_2\text{-cl}(A,s)),s,r)\\ &= f(\mathcal{T}_2\text{-cl}(A,s)).\end{aligned}$$

(2) $\Rightarrow$ (1) Let A be any IF $\mathcal{T}_1$-r-closed set and B any IF $\mathcal{T}_2$-s-closed set in X. Then $A = \mathcal{T}_1\text{-cl}(A,r)$ and $B = \mathcal{T}_2\text{-cl}(A,s)$. Thus by (2), we have

$$\begin{aligned}(\mathcal{U}_1,\mathcal{U}_2)\text{-scl}(f(A),r,s) &\subseteq f(\mathcal{T}_1\text{-cl}(A,r))\\ &= f(A)\\ &\subseteq (\mathcal{U}_1,\mathcal{U}_2)\text{-scl}(f(A),r,s)\end{aligned}$$

and

$$\begin{aligned}(\mathcal{U}_2,\mathcal{U}_1)\text{-scl}(f(B),s,r) &\subseteq f(\mathcal{T}_2\text{-cl}(B,s))\\ &= f(B)\\ &\subseteq (\mathcal{U}_2,\mathcal{U}_1)\text{-scl}(f(B),s,r).\end{aligned}$$

Hence $f(A)$ is IF $(\mathcal{U}_1,\mathcal{U}_2)$-$(r,s)$-semiclosed and $f(B)$ is IF $(\mathcal{U}_2,\mathcal{U}_1)$-$(s,r)$-semiclosed in Y. Therefore f is an IF pairwise (r,s)-semiclosed mapping. □

THEOREM 2.8. *Let $f : (X,\mathcal{T}_1,\mathcal{T}_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a bijective mapping from an ISBTS X to an ISBTS Y and $r,s \in I_0$. Then f is IF pairwise (r,s)-semiclosed if and only if*

$$f^{-1}((\mathcal{U}_1,\mathcal{U}_2)\text{-}scl(B,r,s)) \subseteq \mathcal{T}_1\text{-}cl(f^{-1}(B),r)$$

and

$$f^{-1}((\mathcal{U}_2,\mathcal{U}_1)\text{-}scl(B,s,r)) \subseteq \mathcal{T}_2\text{-}cl(f^{-1}(B),s)$$

for each intuitionistic fuzzy set B in Y.

Proof. Let B be an intuitionistic fuzzy set in Y. Since f is onto, by Theorem 2.7, we obtain

$$\begin{aligned}(\mathcal{U}_1,\mathcal{U}_2)\text{-scl}(B,r,s) &= (\mathcal{U}_1,\mathcal{U}_2)\text{-scl}(f(f^{-1}(B)),r,s)\\ &\subseteq f(\mathcal{T}_1\text{-cl}(f^{-1}(B),r))\end{aligned}$$

and

$$\begin{aligned}(\mathcal{U}_2,\mathcal{U}_1)\text{-scl}(B,s,r) &= (\mathcal{U}_2,\mathcal{U}_1)\text{-scl}(f(f^{-1}(B)),s,r)\\ &\subseteq f(\mathcal{T}_2\text{-cl}(f^{-1}(B),s)).\end{aligned}$$

Since f is one-to-one, we have

$$\begin{aligned}f^{-1}((\mathcal{U}_1,\mathcal{U}_2)\text{-scl}(B,r,s)) &\subseteq f^{-1}(f(\mathcal{T}_1\text{-cl}(f^{-1}(B),r)))\\ &= \mathcal{T}_1\text{-cl}(f^{-1}(B),r)\end{aligned}$$

and

$$\begin{aligned}f^{-1}((\mathcal{U}_2,\mathcal{U}_1)\text{-scl}(B,s,r)) &\subseteq f^{-1}(f(\mathcal{T}_2\text{-cl}(f^{-1}(B),s)))\\ &= \mathcal{T}_2\text{-cl}(f^{-1}(B),s).\end{aligned}$$

Conversely, let A be an intuitionistic fuzzy set in X. Since f is one-to-one, we obtain

$$\begin{aligned}f^{-1}((\mathcal{U}_1,\mathcal{U}_2)\text{-scl}(f(A),r,s)) &\subseteq \mathcal{T}_1\text{-cl}(f^{-1}(f(A)),r)\\ &= \mathcal{T}_1\text{-cl}(A,r)\end{aligned}$$

and

$$\begin{aligned}f^{-1}((\mathcal{U}_2,\mathcal{U}_1)\text{-scl}(f(A),s,r)) &\subseteq \mathcal{T}_2\text{-cl}(f^{-1}(f(A)),s)\\ &= \mathcal{T}_2\text{-cl}(A,s).\end{aligned}$$

Since f is onto, we have

$$\begin{aligned}(\mathcal{U}_1,\mathcal{U}_2)\text{-scl}(f(A),r,s) &= f(f^{-1}((\mathcal{U}_1,\mathcal{U}_2)\text{-scl}(f(A),r,s)))\\ &\subseteq f(\mathcal{T}_1\text{-cl}(A,r))\end{aligned}$$

and

$$\begin{aligned}(\mathcal{U}_2,\mathcal{U}_1)\text{-scl}(f(A),s,r) &= f(f^{-1}((\mathcal{U}_2,\mathcal{U}_1)\text{-scl}(f(A),s,r)))\\ &\subseteq f(\mathcal{T}_2\text{-cl}(A,s)).\end{aligned}$$

Thus by Theorem 2.7, f is an IF pairwise (r,s)-semiclosed mapping. $\square$

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